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Fine arts in dialogue with mathematics. Mutual educational inspirations

Sztuki plastyczne w dialogu z matematyką. Wzajemne inspiracje edukacyjne

Abstract

Aim. The aim of this article is to show, from the perspective of an artist and a teacher, the possibility of integrating mathematical knowledge with visual arts, and above all with architecture, sculpture, and painting in relation to selected works.

Methods and materials. The method of document analysis was used, examples of the analysis of works of art and architectural objects from a historical perspective were indicated in terms of the mathematization of their structure, composition, etc. They can be an inspiration for teachers to search for their own solutions in combining art with mathematics at various educational stages.

Results and conclusion. Understanding how mathematical concepts influence the creation and reception of works of art develops analytical and problem-solving skills

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and stimulates creativity. Students learn not only to apply mathematical principles, but also to understand their practical significance and their impact on the world around them. The combination of mathematics and art has led to the creation of many works of exceptional depth and beauty, which not only delight with their aesthetics, but also stimulate intellectual curiosity. Thus, mathematics and art are two areas of human activity that are in harmony with each other, creating new opportunities for artistic expression, but also for improving mathematical competences, developing the skills of observation, analysis, synthesis, and critical thinking. Integrating an interdisciplinary approach to mathematic education can lead to a profound transformation in the way students perceive and engage with the subject.

Keywords: education, mathematics, fine arts, integration, geometry, pedagogics.

Abstrakt

Cel. Celem niniejszego artykułu jest ukazanie – z perspektywy artysty plastyka i zarazem pedagoga – możliwości integrowania wiedzy matematycznej ze sztukami plastycznymi, a przede wszystkim z architekturą, rzeźbą i malarstwem w odniesieniu do wybranych dzieł. Przedmiotem analizy są wzajemne relacje nauki i sztuki, czyli rzecz o wpływie matematyki na sztuki wizualne.

Metody i materiały. Zastosowano metodę analizy dokumentów, wskazano na przykłady analizy dzieł plastycznych i obiektów architektonicznych w ujęciu historycznym pod kątem matematyzacji ich struktury, kompozycji itd. Mogą one stanowić inspirację do poszukiwania przez nauczycieli własnych rozwiązań w zakresie łączenia sztuki z matematyką na różnych etapach edukacyjnych. Integracja matematyki z analizą sztuki rozwija także zdolności poznawcze i twórcze uczniów.

Wyniki i wnioski. Zrozumienie, jak pojęciamatematyczne wpływają na tworzenie i odbiór dzieł sztuki, rozwija umiejętności analityczne i problemowe, a także stymuluje kreatywność. Uczniowie uczą się nie tylko stosować zasadymatematyczne, ale także rozumieć ich praktyczne znaczenie i wpływ na otaczający ich świat.Połączenie matematyki i sztuki doprowadziło do powstania wielu dzieł o wyjątkowej głębi i pięknie, które nie tylko zachwycają estetyką, ale również pobudzają intelektualną ciekawość. Matematyka i sztuka to dwa obszary działalności ludzkiej, które pozostają ze sobą w harmonii, tworząc nowe możliwości wyrażania artystycznego, ale także podnoszenia kompetencji matematycznych, rozwijania umiejętności obserwacji, analizy, syntezy i krytycznego myślenia. Integracja interdyscyplinarnego podejścia w nauczaniu matematyki może prowadzić do głębokiej transformacji w sposobie, w jaki uczniowie postrzegają ten przedmiot i angażują się w jego naukę.

Słowa kluczowe: edukacja, matematyka, sztuki plastyczne, integracja, geometria, pedagogika.

Introduction

In the modern world, there is a widespread and ever-increasing mathematisation of various spheres of human activity. This process, hidden behind computerisation and informatisation, is often not yet sufficiently realised and noticed. At the same time, to a greater or lesser extent, man functions in the space of art, which is a unique form of activity in perception and performance. According to Jerzy Vetulani (2011), "[...] the research efforts of neurobiologists, educators and art theorists have shown that art, its reception and creation, are immanent and peculiar features of human nature and are factors that activate the entirety of brain activity. Learning to receive art and to produce art develops our cognitive attention and with it all the cognitive aspects of our brain" (p. 13). According to Kinga Czerwinska (2017), the arts "[...] have the power to form a canon of values, sensitise us to problems and build social attitudes, stimulate our senses and give impetus to aesthetic satisfaction" (p. 103).

The dynamics of economic, socio-cultural and civilisational changes make us reflect both on the importance of artistic education in the upbringing and education of children and young people, and on the search for connections between the world of numbers, symbols, the technicisation of various areas of life and the preparation of young people for life in contemporary reality through contact with the visual arts, such as painting, sculpture, architecture, graphics and others. Enabling the integral development of all spheres of children's and young people's personalities requires adopting the stance that mathematics as a field of science and art

> [...] not only are they not opposed to each other, they are similar and complementary. One might even venture to say that science is also an art. It is the art of knowing and building the edifice of human knowledge. The aesthetic value of the theories built-in science and, above all, the aesthetic value of certain mathematical structures is sometimes equal to the beauty of the most outstanding works of art (Misiek, 2010, p. 211).

Conversely, the question of mathematics in art is considered on a scientific basis. In this context, mathematics is treated as a kind of tool to achieve artistic goals. Mutual inspirations of mathematics and art are analysed, with mathematical inspirations in art being more visible, direct and legible (Orlikowski, 2018). The stereotypical view that science and art are opposite fields of activity is becoming a thing of the past. The paradigmatic shifts taking place in contemporary education consist – among other things – in a move away from traditional subject-based teaching to a much broader idea, which has been labelled with the acronym – STEAM (*Science, Technology, Engineering, Art, Mathematics*), in which we also include an art element in mathematics, science, and technology. The arts are treated in education as a full-fledged field, leading to learning about the world and the real development of children and young people (Minchberg, 2018). STEAM education, which is implemented in educational practice from the lowest levels of education (Bojarska-Sokołowska, 2021), allows for the integration of knowledge from different fields of study, and the projects implemented involve children/students searching independently or in teams for specific information in a selected area of education using different sources of knowledge, and searching for practical ways to apply the knowledge gained through research, rather than merely assimilating knowledge from different areas given by the teacher (Trojańska, 2018). The use of the resources, means and tools of the arts in working with children and young people in conjunction with other subjects in school, including mathematics, can become an effective way to support an educational environment that should aim to guide students to be creative and develop critical thinking (Minchberg, 2018) and to experience the world holistically.

It is the task of the author of this paper to show – from the perspective of a visual artist and also an educator – the possibilities of integrating mathematical knowledge with the visual arts, especially with architecture, sculpture, and painting. The examples of analyses of visual arts and architectural objects from the historical perspective of the mathematisation of their structure, composition, etc. cited in the following part of the text may inspire teachers to search for solutions of combining art and mathematics at various educational stages.

Geometry, numbers, and proportions in the visual arts

The coexistence of such seemingly disparate fields and school subjects as visual arts and mathematics in the space of life and education raises the question of the possibility and nature of "dialogue" between them. In today's world, there is constant talk of dialogue and its consequences. Regardless of the fields of knowledge in which it occurs, the need to conduct it in an interdisciplinary manner is emphasised. In a dialogue understood in this way, the role of "interlocutor" may be fulfilled by various cultural messages, such as literature, music, art, film, etc. Therefore, concerning the subject matter addressed in this paper, the aim is to emphasise – in the process of dialoguing visual arts and mathematics – on the one hand, their mutual connections, "common features," on the other hand – to preserve their differences and specificity. In the case of the visual arts, the perception and perception of aesthetic reality are very often limited to the subjective-emotional, in which no rules apply and the only criterion for evaluation is impression. However, the reality is quite different. Just as in the natural world its logical rules apply, so it is in aesthetics. In the case of mathematics, meanwhile, a misconception often prevails in many people's minds that it is an algorithmic discipline of science with reproducibility and schematism at its core. Very often it is colloquially understood only through the prism of school experiences or real-life applications, as the study of numbers or figures. The exemplifications of the connections and "dialogues" between visual arts and mathematics cited below result primarily from the viewing and analysis of original works by the author of the text, who is also a visual artist. They are also described in the literature devoted to the history of art as a whole or the analysis of selected periods in art history (*cf.* Gombrich, 2008; Koch, 2023; Libicki, 2022; Makowiecka, 2008; and others).

In music as well as in the visual arts, i.e., visual arts and architecture, there is a principle of rhythm or repetition of elements. This can already be seen in the cave paintings found in caves such as Altamira and Lascaux. There is a well-known frieze of flowing deer, depicting a herd of these animals crossing a river. In addition to the serial representation of the animals, its creator was able to capture their mental state and the slight differences in the animals' positioning. Strip compositions with rhythmic pictorial forms form the basis of communication in ancient times, appearing in both Egyptian tomb polychromes and temple reliefs as well as hieratic, demotic or hieroglyphic ideograms. It is the simplest way of pictorial communication, informing about the realities of the time, and occurring in chronological order. Typical examples of belt arrangements are the paintings illustrating the feast from Nebamon's tomb and the depictions of ox herders from Horemheb's tomb in the Valley of the Kings. Similar imagery is also found in the mastaba of Ptahotep at Sakkara, as well as in the temple of Queen Hatshepsut at Deir el Bahari and the mastaba of the dignitary Ti. Similarly schematic, heavily geometrised compositions with row arrangements occur simultaneously in Mesopotamian art, examples of which are: The Banner of Ur, dating to 2600 BC, and the Vulture Stella, dating to 2450 BC, as well as reliefs from the palace of King Asurbanipal in Nineveh and Darius the Great in Persepolis. Rhythmisation through the reproduction of figurative arrangements is also found in Byzantine isokephaly, mostly mosaic, created in Ravenna or Constantinople. Rhythmization also occurs in several sculptural representations, most often in the form of reliefs or bas-reliefs, and is closely connected and harmonised with architectural elements in the form of friezes or metopes depicting mythological scenes in ancient Greece. Rhythmic arrangements of all kinds, however, occur primarily as structural elements in buildings erected from prehistoric times to the present day. The most impressive Neolithic menhir avenues in Carnac, Brittany, France, are stacked stone boulders evenly spaced and several metres high over a distance of about one kilometre. Megalithic rhythmic structures also appear as cromlechs in the form of stone circles. The most well-known arrangement of this type is found at Stonehenge in present-day southern England. Also attributed to the Lusatian culture, the settlement at Biskupin is an example of a development of this type, as it consisted of 105 buildings, interconnected, with an area of about 70–80 m^2. Ancient Egyptian architecture developed a form of ceiling beam support in the form of a column surrounding a wreath rhythmically arranged inner part of the temple. The column has a primarily structural significance, but also an aesthetic and decorative one. This ornamentation refers to plants typical of the area: palms, papyrus or lotus. The pinnacle of engineering and art are the Greek columns, which then appear in a variety of stylistic variants, both in Byzantium, Romanism, Gothic, Renaissance, Baroque, Classicism, and in modern times (as cast-iron casts, used most often in the open spaces of railway station buildings).

The oldest of the Greek objects exemplifying mathematisation in art may be the order of the Doric column. It corresponds to the proportions of the male figure, as it has a stocky and massive form. Its substitute is sometimes the sculpted figure of Atlas, supporting the ceiling. The base of the column is formed by the three steps of the temple called stereobate. In turn, it rests on the highest of them, the stylobate. The trunk consists of modules, precisely fitted together. It should be noted that they were all originally made by hand. The height of the module is equal to the length of the radius of the column circle at the base. The shaft is cannelled or fluted, which gives an additional rhythmic effect, while at the same time differentiating the surface texture of this support, resembling the folds of a tunic or chiton. In addition, it has an optical taper, causing it to slim down, occurring at one-third of its height. In the Doric order, 8 to 12 modules were usually used. It is crowned by a head – or capitulum – consisting of a square plate called the abacus and a round stone "cushion" – or echinus. The Doric column supports a beam, consisting of a plain architrave and a frieze, decorated with triglyphs and metopes.

In contrast, the Ionic order present in Greek architecture corresponds to the proportions of the female figure. It is more slender and refined. This column was also sometimes replaced by the figure of the so-called "caryatid," appearing as a sculpture with a chiton wrapped around her body. A side portico with caryatids can be found in the temple of the Erechtheion on the Acropolis. The column itself has a base, or foot, consisting of three rings. The cannelled base is topped by a capitol with volutes – or snails – which, as typically Ionic motifs, refer in their shape to the symbolism of femininity and parenthood. In Greek architecture, the volutes are positioned symmetrically, also resembling a scroll, while in Ancient Rome a diagonal head was used, with the volutes appearing at the four corners of the capitel. The Ionic order has between 12 and 14 modules. The column head supports a beam, consisting of a stepped architrave and a uniform carved frieze. The most slender Greek support is the Corinthian column, which has the most ornate capitol, consisting of stylised acanthus leaves and small volutes. It has between 14 and 16 modules.

In ancient Roman architecture, Greek orders are used more as ornaments rather than structural elements. We therefore distinguish between:

– Roman Doric order,
– Roman-Ionic,
– Roman-Corinthian,
– composite, or combined.

The Romans also used the Tuscan order – often as rhythmic decorative forms in the form of semi-columns or pilasters, occurring between arcades, in the individual storeys of the building, usually four, giving the so-called "pilasteries" of the order. Column systems in Greek temples surrounded the interiors of these buildings with a single or double wreath of columns. These are known as peripteros, dipteros or monopteros. An exception to this is the archaic temple layout – templum in antis – which is the prototype of the portico, as the column columns appear in it as a pair between the antes, or side walls. Greek temples were usually erected on elongated layouts, having a rectangle in the base. The surrounding columns were juxtaposed in proportions – even columns, occurring in the façade and at the back, to odd columns, located on its sides. In the Parthenon temple, for example, there is a ratio of eight columns to seventeen, which, when doubled, gives a total of fifty. In Roman buildings, the base of all structures was a solid arch, which was a form of a semicircle, consisting in fact of clypeal blocks fitted together, the main one being the so-called "keystone" or key, appearing at its apex. A stone system of this type needed solid supports which were pillars. Arches were used in all kinds of buildings, especially public buildings such as aqueducts, theatres, amphitheatres, circuses, odeons, and basilicas. Based on geometric construction, arches were used in various types of vaults. The most commonly realised ones are the barrel vault or the cross vault. They are used later with slight modifications in Byzantine and Romanesque architecture as belt courses or grills or lunette vaults. At the same time, the dome vaulting in different styles has its variations and modifications. In Rome, in addition, the sagalline vault and the so-called "monastery vault" were used. In Byzantine architecture, it is a dome supported by pendentives, i.e., spherical triangles, situated on the four sides of the dome's canopy proper, and in Islam, a dome with characteristic bulbous shapes. A typical feature of Byzantine architecture is the central arrangements, based on the bases of the Greek cross, square, and circle, often interpenetrating each other.

Early Christian architecture, drawing stylistic patterns from ancient buildings, was particularly keen to adopt the basilica, which had previously served as a market hall and even a court. For Christianity after 313 AD, it was and is a temple where services could be held for large numbers of the faithful. Its utilitarian-functional character is based on proportions derived from numbers, which in turn in its concrete elements also have a symbolic meaning. A basilica usually has three naves. The main one is higher and wider than the side ones, a reference to the Holy Trinity. It also has three entrances, where similar proportions are also maintained. The main one is the dominant one and the side ones are smaller. It is finished with a semi-circular apse, where the bishop's throne or tabernacle was originally located. It also features three semi-circular window openings. Inside the building, between the naves, pillars or columns support the whole structure. There are six between the naves, making a total of twelve, corresponding to the number of apostles. The function of the columns, like the pillars, is to support the edifice of the church. The basilican layout is further modified over time by the addition of a transept, or transverse nave, giving a new plan to the church based on the proportions of the Latin cross. In Romanesque architecture, this is enriched in the façade by the addition of two towers flanking the façade, so that the cruciform system is unified, as it were, in a mass, reminiscent of a woman and expressing the idea of the Mother Church. Romanism in the construction of sacred buildings also introduces the so-called "tied system," in which the square of the nave corresponds to the four smaller squares of the side aisles. Bonding also applies to the connected pillars and cross vaults. In Gothic architecture, the basilica systems are retained. However, the hall temple type is introduced, which also has three naves, but of the same height and width. The transept is moved to the central part of the temple, thus lengthening the presbytery, which in turn is enriched with an ambit, or bypass, connected to the so-called "wreath of chapels," the largest and most significant of which is the central Marian chapel, dedicated to the Virgin Mary.

Plato's idea of defining an ideal reality, which has its imperfect reflection in the real one, envisages the existence of weaker entities, i.e., those which, while perfecting themselves internally, constantly ascend, consequently reaching their highest level, about the categories of truth, goodness and beauty. The visual schematic representation of this belief is the form of an equilateral triangle. Its analysis was dealt with by, among others, the ancient philosopher Pythagoras. It was considered perfect because it has all sides equal, closing into a single whole. Each side can be divided into three equal segments and then connected inside the figure by parallel lines. This division results in an arrangement of nine inner equilateral triangles, one-third smaller in scale, whose sum of vertices gives the number ten, which in turn allows any arithmetic operation to be performed, and these divisions can be multiplied indefinitely. In the view of the ancients, this figure reflects the logic of the order of the universe perceived as a microcosm and a macrocosm. A composition of this type of

> [...] is of a closed character, with the figure-defining its framework being a triangle, and it can be framed by the form of this figure or only contained within the triangle. This way of organising space was popular in antiquity, especially in the Hellenistic period, when many sculptural works such as the "Laocon Group" and the "Pharnesian Bull" were composed on the triangular principle. During the Renaissance,

it was fondly used by Leonardo da Vinci (typical examples of such a composition are the "Mona Lisa" and "Saint Anne Alone") and Raphael Santi, who based his countless Madonnas on this scheme. The aforementioned compositional motif was also quite popular during the Baroque period. The equilateral triangle, whose lines are a unity of three equal sizes, became a symbol of the triune God. Since in this triangle, each side is a mediating and unifying element for the other two, this trinity appears, as a unity. This becomes even more apparent if the triangle describes a circle (Forstner, 1990, p. 59).

Also connected to the concept of the search for the ideal in geometric forms is the symbolism of the circle, which is a graphic representation of infinity. As Dorothea Forstner (1990) writes,

> [...] it was already believed in ancient times that the circle, as a line returning to itself, all points of which are equally distant from the centre, is not only the simplest but also the most perfect figure. In a circle, there is nothing "before" or "beyond," nothing, neither greater nor lesser. It combines the greatest stillness with the most tense strength and is therefore an image of completeness and perfection. Since it has neither a beginning nor end, it is the image of eternity. All other geometrical figures can be derived from the circle (p. 65).

Since prehistoric times, the figure has been a symbol of feminine beauty, associated with the transmission of life and parenthood. Hence, in the Neolithic and Bronze Age, there were many signs reflecting the shape of a star inscribed in a circle, spiral or rosette form. This symbolism is later adopted by Christianity, as a reference to the figure of the Virgin Mary (e.g., in many facades of Gothic cathedrals in the form of a rose flower). In aesthetics, the circle is one of the essential compositional arrangements. Ernest Zawada (2006) writes:

> A composition in a circle consists of composing the individual elements in such a way that they form a figure similar to a circle. Such a composition may be surrounded by a circle or present an element that has the form of a circle or oval. Very often it can take on an ornamental character, forming a rosette, the essence of which is central arrangement and symmetry. Examples of this type of composition are floral, solar or star motifs. The composition in a circle was popular during the Renaissance; it is even referred to by the special term "Tondo." It was used not only in painting but also in architecture (p. 58).

Centrifugal compositions, preserving the circular shape, are the domain of all kinds of handicraft forms. Currently, one of the most distinctive is Koniakowska lace, whose tradition of manufacture is over a century old. According to accounts, the skill of making this lace was originally presented to girls in school classes during so-called "handwork." In the Austro-Hungarian monarchy, of which Cieszyn Silesia was a part, traditions and courtly duties were transferred to the educational sphere. Koniakowska lace has a circular shape and consists of small, separately made elements. These are usually interpretations of floral motifs taken from nature, hence their regional dialect name "róziczki." They are all "heked" with crochet. Each element is separately "counted," which means that the completion of one motif requires the thread weaver to make the appropriate number of crochet stitches. In turn, all the various finished elements, which are also counted, must fit together, as this is the only way in which their steady orderly growth is possible. There is also the possibility that larger-scale lace-making realisations are made not by one person, but several. Hence the initiative to create the record-breaking largest lace in the world in Koniaków. It is several metres in diameter and is an authentic work of handicraft and decorative art.

Ornament, or ornamentation, based on the form of a circle, is one of the most typical, repetitive compositional arrangements of a decorative nature. It is most often formed by floral-organic, figural-zoomorphic and geometric-abstract motifs. They are most common in Islamic and Far Eastern art and express the idea of the order of the universe. In the Indian mandala, this is given a special expression as sand particles are used. The process of its creation is based on harmonic development, only to be destroyed as a result. The organic nature of the mandala's motifs symbolises the natural world, which must be logically tamed, cultivated as a garden, or built as "urbis" and "orbis" at the same time, inscribed e within the framework of a diagrammatic geometry, of interpenetrating circles and squares. A similar principle of ordering the reality of nature and combining it with elements of palace architecture can be seen in the art of the European Baroque. Examples are the garden layouts of Versailles, Potsdam or Wilanów, designed on the axis of symmetry.

A graphic representation of another geometric figure, the square, is depicted in Leonardo da Vinci's well-known sketch *Homo quadratus*, which presents the figure of a human being inscribed in a circle and square. Man, the humanist, is inscribed in the logic of the universe. However, the square as a pure aesthetic figure did not exist until the beginning of the 20th century. It was Kazimierz Malewicz – a Pole by origin, working in tsarist and then Soviet Russia – who was the first to create an autonomous work depicting the "feeling of non-objectivity," i.e., a black square on a white background. This gave rise to the development of his concept of suprematism, i.e., the domination of the human intellect in art, expressed as pure and "cool" geometric abstraction, subsequently transposed in many ways in various stylistic conventions: neo-plasticism, constructivism, art déco and minimalism, both in the visual arts and in design, the consequence of which is the propagation of this idea (through the Bauhaus) right up to the present day.

An example of geometrisation in art is also the so-called “golden ratio principle.” The golden ratio, otherwise known as the “golden cut” of segment and plane, is a compositional principle originating in ancient Greece. It reflects the search for ideal proportions and perfect harmony. The principle consists of dividing a section into two parts in such a way that the ratio of the smaller part to the larger part is equal to the ratio of the larger part to the whole section. It is expressed by the number approximately 0.618 and, based on this, a sequence of numbers with ratios similar to the golden ratio can be arranged in the following order: 3–5–8–13–21. The golden division is the accent of the composition, to which all its other elements are subordinated. The golden division aroused great interest during the Renaissance. At the time, it was referred to as the gateway to harmony or divine proportion. The term originated from the title of Luca Paccioli’s work *Divina proportione*. Renaissance artists used the golden division in the canon of the human figure, in composition or the shapes of canvases. A renewed interest in the golden division came in the 19th century when it was noted that it was a universal proportion of natural creations and perfect human works. In the twentieth century, this issue was addressed by Le Corbusier, among others, and in Poland by Władysław Strzemiński (Zawada, 2006).

An example of the mathematisation in sensory perception of elements of nature that do not have a typically geometric structure, such as invertebrates, clouds, sea waves, fern leaves, shells, broccoli, etc., are fractals – objects whose fragments look like a whole in close-up. They can be enlarged at will by repeating modules as well as certain mathematical operations. They are infinitely self-similar objects that can take on a variety of forms through multiplication. As unusual sets, they form a new kind of so-called fractal geometry. They can be obtained by repeating the same operation. Thanks to their self-similarity, fractals help in the design of models of geometrical structures (Olek, 2018).

Vector and raster images are frequently used in advertising design today. Vector images rely on the geometric description of points and curves, occurring in a coordinate system. Vector files are used as schematics and logos. They are used when creating websites. In raster graphics, images must be saved as bitmaps, where individual pixels have many parameters assigned to them. Raster graphics are used to create photorealistic images in the types of concept art, digital art, and digital painting.

Examples of the connections between art and mathematics can also be found in the lettering constructions derived from the synthesis of pictorial signs. At present, these are divided into typically geometric and non-geometric. The Far Eastern character system is of a more free repetitive arrangement, derived from pictorial notations, originally created by hand as traces of ink and brush left on paper. Alphabetic writings, both Latin and Cyrillic, derived from Phoenician characters, are based primarily on geometric constructions. The Latin alphabet, the so-called “majuscule,” (i.e., capital letters) was based on the simplest geometric systems, such as the vertical, horizontal, oblique, circle, arc, and ellipse. The basic measure in their creation is the modulus of the square and the grid of grids, allowing proportions to be defined and enlarged or reduced in scale. The Roman majuscule, the so-called “capitula,” has a particularly refined aesthetic expression, as it is a two-element script with “serifs,” or ornaments usually crowning their ends, structurally derived from the circular form. The proportions of all of them follow the shape of a square, but the individual ones occupy half its width, three-quarters or all of it, while their height remains the same. Some of them are created by combining horizontal-vertical and diagonal arrangements, e.g., A, E, F, H, K, L, M, N, V, and Z. The letters: B, C, D, G, O, P, R, or S are examples of the use of vertical and oblique, and above all circular and elliptical arrangements. It is worth mentioning that the finesse of this script is based on the principles of harmony, proportion, symmetry, and asymmetry. Each letter has its own internal dynamics, and their internal and external “lights” allow them to be better perceived and read.

The combination of mathematics and art has led to the creation of many works of exceptional depth and beauty, which not only delight aesthetically but also stimulate intellectual curiosity. In this way, mathematics and art are two areas of human activity that are in harmony with each other, creating not only new opportunities for artistic expression but also for enhancing mathematical competence and developing skills of observation, analysis, synthesis, and critical thinking.

The interrelation of visual arts and mathematics as inspiration in the educational process

The examples cited earlier for analysing works of art and architectural objects show how mathematics and art are interrelated. In works of art, one can find numbers, points, lines, angles, plane and spatial figures, symmetry, perspective, etc., which are the subject of learning in integrated education and the older grades of primary school as well as in secondary schools. Pupils realise that the essence of mathematics is not only contained in detached concepts, numbers and calculations. They see the connections between the mathematics they learn in lessons and social and cultural realities. This can have a positive impact on their motivation to learn the subject, which all too often causes fear and anxiety in learners due to its level of abstraction and difficulty. Research conducted on students’ attitudes towards mathematics (Dąbrowski, 2013; Devine, Dowker, Fawcett, & Szücs, 2012; Erdogan, Kesici, 2010) shows that many of them believe that they will never be able to understand mathematics, at most, they can learn it in such a way as to give the illusion that they understand it. A significant proportion of schoolchildren believe that mathematics is just a set of algorithms or an unnecessary school subject with overly complicated content that only people working as, for example, accountants should know. Many students show a fear of mathematics, which results in their increased dislike of the subject. Moreover, Urszula Oszwa and Magdalena Bakun (2016) highlight that aversion to mathematics in some environments is generational in frequency and creates a barrier against truly understanding this area of knowledge.

The key to changing students' attitudes towards mathematics as a subject can be found in a properly understood integration of its content with other subjects and fields of knowledge, including the visual arts, which is not apparent, but consists of conducting interdisciplinary analyses in both fields, to expose students to the widest possible range of interpretative and explanatory perspectives and, as a result, integrate their knowledge. As Dorota Klus-Stańska and Marzenna Nowicka (2005) note, "[...] integrated knowledge constitutes a dynamic system [...]. It gives the individual a sense of cognitive control over their thinking and acting [...]. Integrated knowledge constructs and reconstructs the individual's ways of thinking, rather than merely replenishing the memory resources of information" (p. 205).

Teachers expect to be inspired by the connections between mathematics and the visual arts. Publications popularising specific methodological solutions in this area are appearing on the publishing market, such as, for example, the book *Szkice o geometrii i sztuce. Sztuka konstrukcji geometrycznych* [Sketches on geometry and art: The art of geometric constructions] (Majewski, 2013), in which the author – Mirosław Majewski – prepares the reader to understand examples of Islamic or Gothic art and to create their works of art with geometric origins. He also discusses various geometric constructions on its pages, useful both in geometry lessons at school and in artistic work. On the other hand, Maja Kramer (2018) in *Matematyka jest wszędzie. Rodzinne przygody z matematyką* [Maths is everywhere. Family adventures with mathematics] show parents and teachers how to discover friendly mathematics in everyday life actively.

For example, one of the main tasks of early childhood education – according to the currently binding core curriculum – is to provide children with access to valuable sources of information in the context of their development and to support cognition of national culture, perception of art and development of the need to co-create it to the extent adequate to the child's developmental stage, as well as fostering the need to learn about art. Early childhood education, implemented by design as integrated education, offers great opportunities for "dialoguing" mathematics with content in art education. The content and scope of the mathematical achievements included in the core curriculum about the understanding of spatial relations and size features, the understanding of numbers and their properties, the use of numbers, and especially concerning the understanding of geometrical concepts and the application of mathematics in life situations and other areas of education, fully corresponds to the aims of visual arts education in terms of visual perception, observation and experience, creative expression and the reception of visual arts. Naming the fields of visual arts, recognising and naming the basic genres of painting and graphic works can be accompanied by mathematical activity, the essence of which is contained in the previously mentioned actions on numbers and mathematical concepts. Children's exploration of selected works of art can involve identifying shapes, sizes, and proportions, the position of objects and elements, and detecting differences and similarities in the appearance of the same object depending on the position and change of position of the person looking at the object, etc. The child's contact with different fields of art can be accompanied by:

– initiating activities involving the recognition of geometrical figures in the natural surroundings, noticing symmetry in applied arts, identifying and presenting the relative position of objects on a plane and in space,
– comparing objects in terms of a distinguished size characteristic,
– using the concepts of vertical, horizontal and slanting,
– making mathematical calculations.

There are also great opportunities for integrating mathematics and arts education through the use of the computer and other digital devices for visual programming by pupils in primary school grades I–III. Through an integrated approach to education, young children naturally engage in early exploration of their environment through hands-on, multisensory, and creative experiences, developing their curiosity, inquisitiveness, critical thinking, and problem-solving skills (White, 2004).

At the beginning of children's education, already in kindergarten and especially in early childhood education, it is crucial to lay the foundations for their further development. At this stage of learning, children must have access to valuable sources of information that support their learning about the national culture. In this way, pupils begin to form their identity, which is essential for their later functioning in society. One of the fundamental aims of early childhood education is to develop in children the need to perceive art and to contribute to its creation. Practical artistic activities enable children to express themselves and to understand the world around them. Forms of artistic expression must be adapted to the developmental stage of pupils, which increases their engagement and motivation to learn. Early childhood education also provides opportunities to integrate mathematical and artistic content. Skills related to the understanding of geometric shapes and spatial relations can be developed through artistic activities, which fosters creativity and enriches children's experiences. The use of mathematics in the context of art allows for learning through play, which is extremely important at this stage of education.

According to the core curriculum in art education for pupils in the older grades of primary school, all knowledge of art theory and its history is only a supplement and learning basis for the art activities undertaken, but not only. In the case of integrating fine arts with mathematics, during the process of learning about the fields of fine arts (painting, sculpture, graphic arts, architecture, etc.), creating various compositional arrangements on the plane and in space, determining the correct proportions of individual compositional elements, creating arrangements and spatial arrangements from various elements, etc., pupils have the opportunity to learn and better understand mathematical concepts and actions, which allows them to improve their abstract thinking and, consequently, to learn how to carry out reasoning and correct conclusions in new and unusual situations. In this context, the analysis of artistic creations can be combined with mathematical activities involving more than just calculations. An example of interdisciplinary mathematics teaching is the observation of geometry in architecture, which connects mathematical knowledge with the surrounding world. Pupils have the opportunity to recognise and name a variety of geometrical figures such as points, lines, rays, and segments, which lays the foundation for further understanding of geometry. Through the analysis of architectural elements, children learn to identify straight lines, and perpendicular and parallel segments, which enables them to see these shapes in practice, for example on buildings or bridges. During the lessons, pupils also have the chance to point out the arms and vertex in an angle, which allows for a deeper understanding of the concept of angles. Introducing the concepts of: right angles, acute angles and obtuse angles, as well as comparing them, develops children's analytical skills and allows them to see the differences between angles in different architectural contexts. In addition, pupils learn about obtuse and adjacent angles, which enriches their understanding of the relationship between figures.

Observing architecture also provides opportunities to recognise straight prisms, pyramids, cylinders, cones and circles, which is particularly inspiring in the context of familiar buildings. Pupils can see how these geometrical figures are used in practice, which gives their learning a real and tangible dimension. This interdisciplinary approach to teaching mathematics develops in children not only mathematical skills but also creativity and the ability to see connections between different areas of knowledge. By combining mathematics with architecture, children become active researchers of the world around them, which strengthens their interest in science and their critical thinking skills. By analysing the shapes of buildings and other artistic objects, pupils understand the influence of mathematics on architectural design in historical and contemporary terms. In addition, they learn about the cultural and social contexts of the works analysed. In turn, mathematics can become an important tool in the artistic process expressed in the creation of artworks, compositions, fractals and the discussion of related geometrical elements. The integration of mathematics with visual arts has numerous benefits in the development of pupils. Through such activities, children have the opportunity to develop their creativity and spatial imagination. The hands-on approach also encourages the development of manual skills, which is extremely important at the early school stage. In addition, this combination promotes logical thinking, enabling pupils to better understand the relationships between different concepts. Finally, engaging in artistic activities promotes improved memory and concentration, which has a positive impact on their overall cognitive development.

Conclusion

The integration of visual arts with mathematics builds the field for mutual numerous inspirations, which are of great importance in education. The collaboration of the two disciplines allows students to see not only the theoretical aspects of mathematics but also its practical application in artistic reality. The cross-fertilisation of mathematical and artistic content creates a new quality of teaching in which pupils develop not only analytical skills but also creativity, spatial imagination, and manual skills.

Mathematics – often regarded as an abstract and theoretical discipline – takes on a new dimension when it is integrated with other disciplines, such as the visual arts. This combination not only enables students to see mathematics in a cultural and social context but also facilitates their understanding of its practical applications in everyday life. Visual arts offer a rich source of examples that illustrate the application of mathematical concepts such as proportion, symmetry, geometry or perspective. Analysing works of art that use these principles allows students to see mathematical structures and patterns in an artistic context. Examples include the use of the golden ratio principle in Renaissance painting or the study of symmetry in Gothic architecture. This approach not only facilitates the understanding of mathematics but also develops students' aesthetic skills, shaping their ability to evaluate and interpret beauty and artistic values.

From a pedagogical perspective, integrating visual arts with mathematics can also contribute to the holistic development of students. Instead of treating mathematics as a separate, isolated subject, it can be integrated with art teaching, which fosters more complex and integrated knowledge structures. Such an interdisciplinary teaching model supports the development of critical thinking skills, analysis, and synthesis of information, which is crucial in today's world. Visual arts are fundamental to the full intellectual development of students. Neglecting this aspect in education means limiting their opportunities to develop their creativity, analytical abilities and aesthetic skills. Therefore, implementing interdisciplinary approaches in mathematics teaching that incorporate the visual arts becomes a crucial element of modern education. This not only enables students to better understand and apply mathematics but also develops their artistic and cultural skills, preparing them to function in a complex, integrated world.

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